ELECTRIC CHARGES AND FIELDS

Charging By Induction

Electrostatics

Branch of science that deals with the study of forces, fields, and potentials arising from the static charges

Electric Charge

- In 600 B.C., the Greek Philosopher Thales observed that amber, when rubbed with wool, acquires the property of attracting objects such as small bits of paper, dry leaves, dust particles, etc.
- This kind of electricity developed on objects, when they are rubbed with each other, is called frictional electricity.
- The American scientist Benjamin Franklin introduced the concept of positive and negative charges in order to distinguish the two kinds of charges developed on different objects when they are rubbed with each other.
- In the table given below, if an object in the first column is rubbed against the object given in second column, then the object in the first column will acquire positive charge while that in second column will acquire negative charge.

I	II
Woollen cloth	Rubber shoes
Woollen cloth	Amber
Woollen cloth	Plastic object
Fur	Ebonite rod
Glass rod	Silk cloth

- Electric charge − The additional property of protons and electrons, which gives rise to electric force between them, is called electric charge. Electric charge is a scalar quantity. A proton possesses positive charge while an electron possesses an equal negative charge (where $e = 1.6 \times 10^{-19}$ coulomb).
- Like charges repel each other whereas unlike charges attract each other.
- A simple apparatus used to detect charge on a body is the gold-leaf electroscope.

Conductors and Insulators

Conductors

- The substances which allow electricity to pass through them easily are called conductors.

  Example − All the metals are good conductors.
- Conductors have electrons that can move freely inside the material.
- When some charge is transferred to a conductor, it readily gets distributed over the entire surface of the conductor.
- When a charged body is brought in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through the connecting conductor (such as our body). This process is known as earthing.

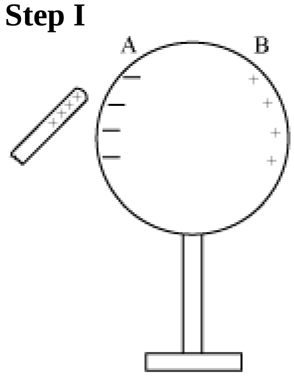
Insulators

- The substances which do not allow electricity to pass through them easily are called insulators.
- Most of the non-metals such as porcelain, wood, nylon, etc. are examples of insulator.
- If some charge is put on an insulator, then it stays at the same place.

Charging By Induction

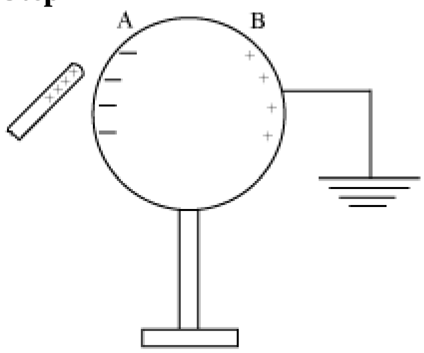
A conductor may be charged permanently by induction in the following steps.

Step I



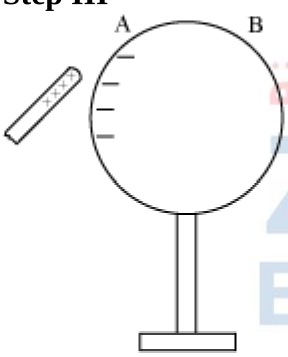
To charge a conductor AB negatively by induction, bring a positively charged glass rod close to it. The end A of the conductor becomes negatively charged while the far end B becomes positively charged. It happens so because when positively charged glass rod is brought near the conductor AB, it attracts the free electrons present in the conductor towards it. As a result, the electron accumulates at the near end A and therefore, this end becomes negatively charged and end B becomes deficient of electrons and acquires positive charge.

Step II



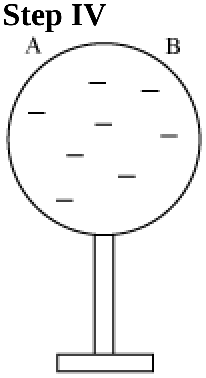
The conductor is now connected to the earth. The positive charges induced will disappear. The negative induced charge on end A of the conductor remains bound to it due to the attractive forces exerted by the positive glass rod.

Step III



The conductor is disconnected from the earth keeping the glass rod still in its position. End A of the conductor continues to hold the negative induced charge.

Step IV



Finally, when the glass rod is removed, the negative induced charge on the near end spreads uniformly over the whole conductor.

Electroscope

Coulomb's Law & Forces between Multiple Charges

Basic Properties of Electric Charges

- Additivity of charges − The total electric charge on an object is equal to the algebraic sum of all the electric charges distributed on the different parts of the object. If q_1, q_2, q_3, … are electric charges present on different parts of an object, then total electric charge on the object, $q = q_1 + q_2 + q_3 + \ldots$
- Charge is conserved − When an isolated system consists of many charged bodies within it, due to interaction among these bodies, charges may get redistributed. However, it is found that the total charge of the isolated system is always conserved.
- Quantization of charge − All observable charges are always some integral multiple of elementary charge, e (= $\pm 1.6 \times 10^{-19}$ C). This is known as quantization of charge.

Coulomb's Law

- Two point charges attract or repel each other with a force which is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = K\frac{q_1 q_2}{r^2} \qquad \text{(i)}$$

Where, $K = \frac{1}{4\pi\varepsilon_0}$ [In SI, when the two charges are located in vacuum]

ε_0 − Absolute permittivity of free space = 8.854×10^{-12} $C^2 N^{-1} m^{-2}$

$$\therefore \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\times 3.14\times 8.854\times 10^{-12}} = 9\times 10^9\ \mathrm{Nm^2C^{-2}}$$

We can write equation (i) as

$$F_{\text{vac}} = 9\times 10^9 \times \frac{q_1 q_2}{r^2}$$

- The force between two charges q_1 and q_2 located at a distance r in a medium may be expressed as

$$F_{\text{med}} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

Where ε – Absolute permittivity of the medium

$$\frac{F_{\text{vac}}}{F_{\text{med}}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon}\cdot\frac{q_1 q_2}{r^2}} \frac{\varepsilon}{\varepsilon_0}$$

The ratio $\frac{\varepsilon}{\varepsilon_0}$ is denoted by ε_r, which is called relative permittivity of the medium with respect to vacuum. It is also denoted by k, called dielectric constant of the medium.

$$\therefore k(\text{or } \varepsilon_r) = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{\text{vac}}}{F_{\text{med}}}$$

$\varepsilon = k\varepsilon_0$

$$\therefore F_{\text{med}} = \frac{1}{4\pi k\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Coulomb's Law in Vector Form

Consider two like charges q_1 and q_2 present at points A and B in vacuum at a distance r apart.

According to Coulomb's law, the magnitude of force on charge q_1 due to q_2 (or on charge q_2 due to q_1) is given by,

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \qquad \text{...(i)}$$

Let

$\hat{r}_{21}$ – Unit vector pointing from charge q_1 to q_2

$\hat{r}_{12}$ – Unit vector pointing from charge q_2 to q_1

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{21}$$ [$\because$ $\vec{F}_{12}$ is along the direction of unit vector $\hat{r}_{21}$] …(ii)

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12}$$ [$\because$ $\vec{F}_{12}$ is along the direction of unit vector $\hat{r}_{12}$] …(iii)

$\therefore \hat{r}_{21} = -\hat{r}_{12}$

$\therefore$Equation (ii) becomes

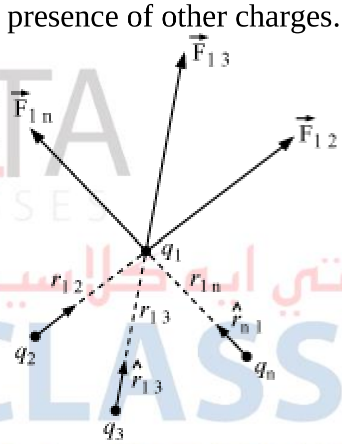
$$\vec{F}_{12} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad \text{...(iv)}$$

On comparing equation (iii) with equation (iv), we obtain

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Forces between Multiple Charges

Principle of superposition − Force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges.



Consider that n point charges q_1, q_2, q_3, … q_n are distributed in space in a discrete manner. The charges are interacting with each other. Let the charges q_2, q_3, … q_n exert forces $\vec{F}_{12}, \vec{F}_{13}, \ldots \vec{F}_{1n}$ on charge q_1. Then, according to principle of superposition, the total force on charge q_1 is given by,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots + \vec{F}_{1n} \qquad \text{...(i)}$$

If the distance between the charges q_1 and q_2 is denoted as r_{12}; and $\hat{r}_{21}$ is unit vector from charge q_2 to q_1, then

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21}$$

Similarly, the force on charge q_1 due to other charges is given by,

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_3}{r_{13}^2}\hat{r}_{31}$$

$$\vec{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_n}{r_{1n}^2}\hat{r}_{n1}$$

Substituting these in equation (i),

$$\therefore \vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{31} + \ldots + \frac{q_1 q_n}{r_{12}^2}\hat{r}_{n1}\right)$$

Electric Field & Continuous Charge Distribution

Electric Field − It is the space around a charge, in which any other charge experiences an electrostatic force.

Electric Field Intensity − The electric field intensity at a point due to a source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

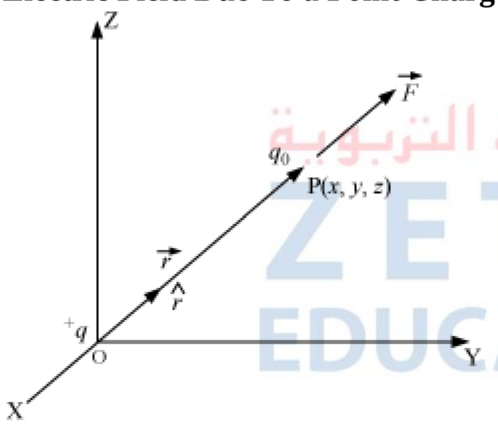
Where,

$\vec{E} \rightarrow$ Electric field intensity

$\vec{F} \rightarrow$ Force experienced by the test charge q_0

Its SI unit is NC^{-1}.

Electric Field Due To a Point Charge



We have to find electric field at point P due to point charge $+q$ placed at the origin such that $\overrightarrow{OP} = \vec{r}$

To find the same, place a vanishingly small positive test charge q_0 at point P.

According to Coulomb's law, force on the test charge q_0 due to charge q is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}\hat{r}$$

If $\vec{E}$ is the electric field at point P, then

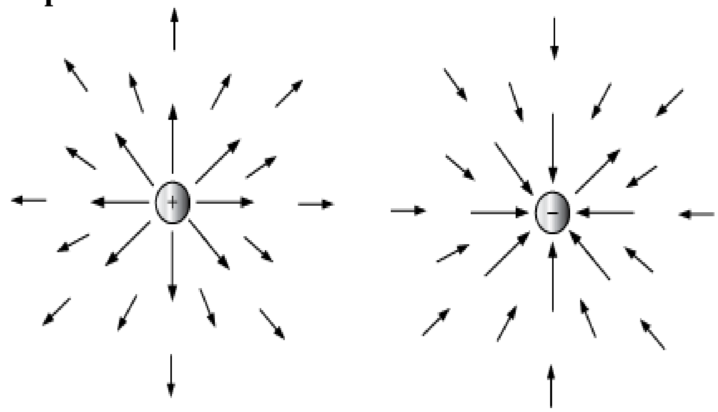
$$\vec{E} = \underset{q_0\to 0}{\mathrm{Lt}}\frac{\vec{F}}{q_0} = \underset{q_0\to 0}{\mathrm{Lt}}\left(\frac{1}{q_0}\cdot\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}\right)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^3}\vec{r} \qquad \ldots(i)$$

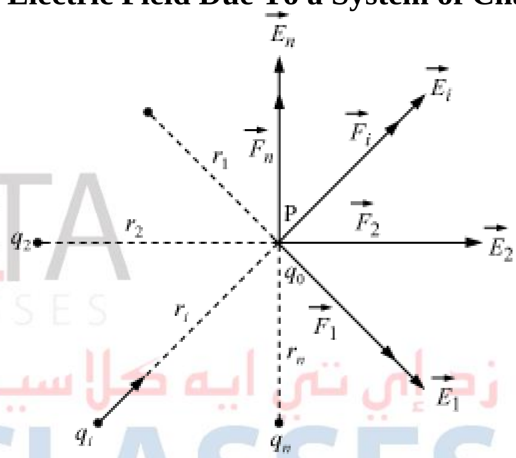
The magnitude of the electric field at point P is given by,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

Representation of Electric Field



Electric Field Due To a System of Charges



Consider that n point charges q_1, q_2, q_3, … q_n exert forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots \vec{F}_n$ on the test charge placed at origin O.

Let $\vec{F}_i$ be force due to i^{th} charge q_i on q_0. Then,

$$\vec{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_0}{r_i^2}\hat{r}_i$$

Where, r_i is the distance of the test charge q_0 from q_i

The electric field at the observation point P is given by,

$$\vec{E}_i = \underset{q_0\to 0}{\mathrm{Lt}}\frac{\vec{F}_i}{q_0} = \underset{q_0\to 0}{\mathrm{Lt}}\frac{1}{q_0}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i q_0}{r_i^2}\hat{r}_i\right)$$

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{r}_i \qquad \ldots(i)$$

If $\vec{E}$ is the electric field at point P due to the system of charges, then by principal of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots + \vec{E}_n = \sum_{i=1}^{n}\vec{E}_i$$

Using equation (i), we obtain

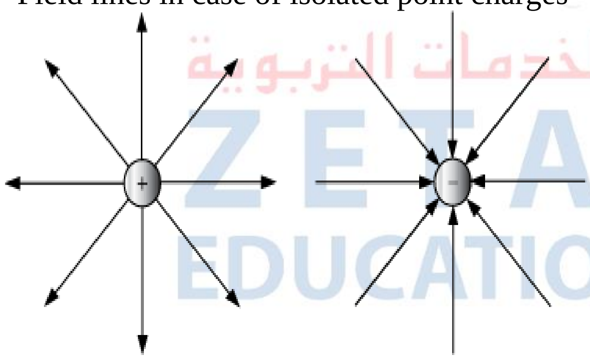
$$\vec{E} = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_i}{r_i^2} \hat{r}_i$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i \qquad \text{...(ii)}$$

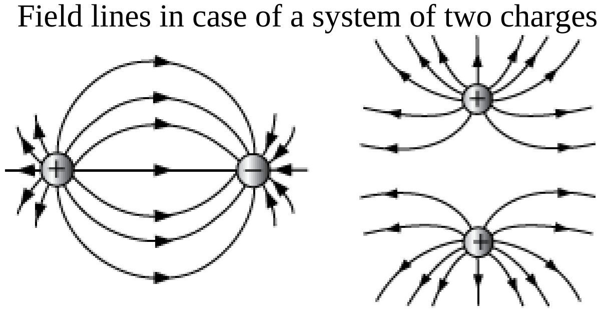
Electric Field Lines

An electric line of force is the path along which a unit positive charge would move, if it is free to do so.

- **Properties of Electric Lines of Force**
- These start from the positive charge and end at the negative charge.
- They always originate or terminate at right angles to the surface of the charge.
- They can never intersect each other because it will mean that at that particular point, electric field has two directions. It is not possible.
- They do not pass through a conductor.
- They contract longitudinally.
- They exert a lateral pressure on each other.
- **Representation of Electric Field Lines**
- Field lines in case of isolated point charges

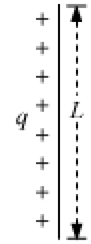


- Field lines in case of a system of two charges



Continuous Charge Distribution

- **Linear charge density** − When charge is distributed along a line, the charge distribution is called linear.



$$\lambda = \frac{q}{L}$$

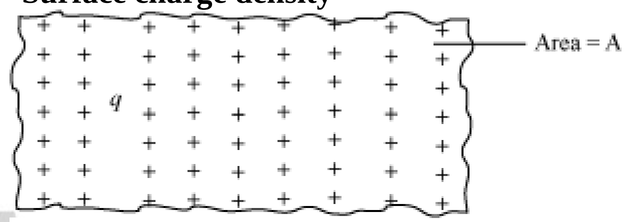
Where,

$\lambda \rightarrow$ Linear charge density

$q \rightarrow$ Charge distributed along a line

$L \rightarrow$ Length of the rod

- **Surface charge density**



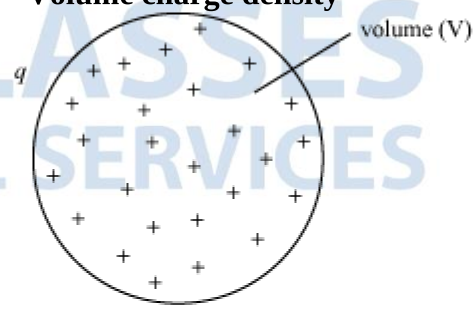
$$\sigma = \frac{q}{A}$$

Where,

$\sigma \rightarrow$ Surface charge density

$q \rightarrow$ Charge distributed on area A

- **Volume charge density**



$$\delta = \frac{q}{V}$$

Where,

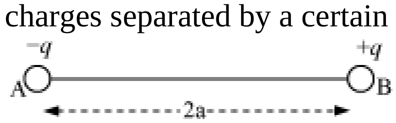
$\delta \rightarrow$ Volume charge density

$V \rightarrow$ Volume of the conductor

$q \rightarrow$ Charge on conductor

Electric Dipole & Dipole in a Uniform External Field

Electric Dipole − System of two equal and opposite charges separated by a certain distance

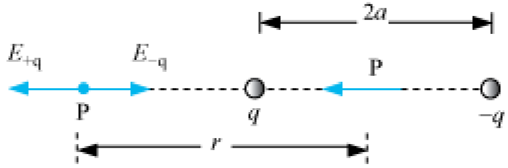


Electric Dipole Moment − Product of either of the charges and the length of the electric dipole

$\vec{p} = q(\overrightarrow{2a})$

Its direction is same as that of $\overrightarrow{2a}$.

Electric Field on Axial Line of an Electric Dipole



Let P be at distance r from the centre of the dipole on the side of charge q. Then,

$$E_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{p}$$

Where, $\hat{p}$ is the unit vector along the dipole axis (from −q to q). Also,

$$E_{+q} = -\frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{p}$$

The total field at P is

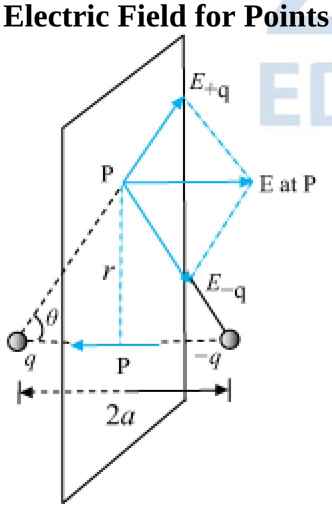
$$E = E_{+q} + E_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p} = \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2}$$

For $r >> a$

$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{p} \qquad (r >> a)$$

$$E = \frac{2p}{4\pi\varepsilon_0 r^3} \qquad \left[\because \vec{p} = q\times\overrightarrow{2a}\hat{p}\right]$$

Electric Field for Points on the Equatorial Plane



The magnitudes of the electric field due to the two charges $+q$ and $-q$ are given by,

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2} \qquad \text{...(i)}$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2} \qquad \text{...(ii)}$$

$$\therefore E_{+q} = E_{-q}$$

The directions of E_{+q} and E_{-q} are as shown in the figure. The components normal to the dipole axis cancel away. The components along the dipole axis add up.

∴ Total electric field

$E = -(E_{+q} + E_{-q})\cos\theta\,\hat{p}$ [Negative sign shows that field is opposite to $\hat{p}$]

$$E = -\frac{2qa}{4\pi\varepsilon_0 (r^2+a^2)^{\frac{3}{2}}}\hat{p} \qquad \text{...(iii)}$$

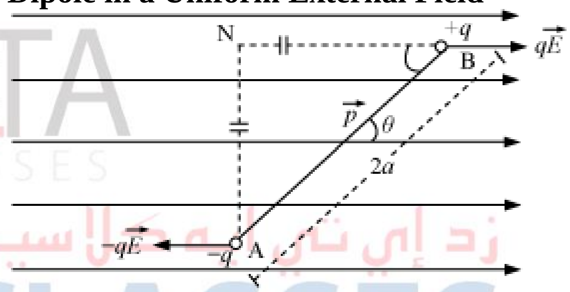
At large distances ($r >> a$), this reduces to

$$E = -\frac{2qa}{4\pi\varepsilon_0 r^3}\hat{p} \qquad \text{...(iv)}$$

$$\because \vec{p} = q\times\overrightarrow{2a}\hat{p}$$

$$\therefore E = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3} \quad (r >> a)$$

Dipole in a Uniform External Field



Consider an electric dipole consisting of charges $-q$ and $+q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with electric field.

Force on charge $-q$ at $A = -q\vec{E}$ (opposite to $\vec{E}$)

Force on charge $+q$ at $B = q\vec{E}$ (along $\vec{E}$)

Electric dipole is under the action of two equal and unlike parallel forces, which give rise to a torque on the dipole.

τ = Force × Perpendicular distance between the two forces

$\tau = qE\,(AN) = qE\,(2a\sin\theta)$

$\tau = q(2a)\,E\sin\theta$

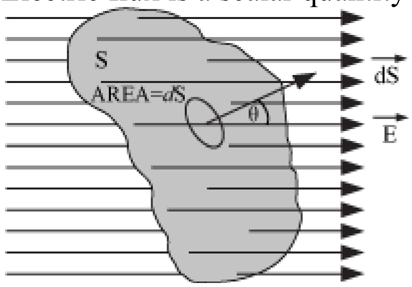
$\tau = pE\sin\theta$

$\therefore \vec{\tau} = \vec{p}\times\vec{E}$

Electric Flux & Gauss Law

The electric flux, through a surface, held inside an electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.

Electric flux is a scalar quantity and is denoted by Φ.



$$\phi = \int_s \vec{E}\cdot\vec{ds} = \int_s E_n\, ds$$

SI unit − $Nm^2\, C^{-1}$

Gauss Theorem

It states that the total electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed.

$$\phi = \frac{q}{\varepsilon_0}$$

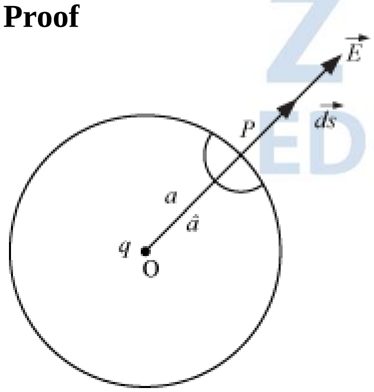
However,

$$\phi = \oint_s \vec{E}\cdot\vec{ds}$$

∴Gauss theorem may be expressed as

$$\boxed{\oint_s \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}}$$

Proof



Consider that a point electric charge q is situated at the centre of a sphere of radius 'a'.

According to Coulomb's law,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}\hat{a}$$

Where, $\hat{a}$ is unit vector along the line OP

The electric flux through area element $\vec{ds}$ is given by,

$$d\phi = \vec{E}\cdot\vec{ds}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\hat{a}\cdot\vec{ds}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}(\mathrm{i})(ds)\cos 0°$$

$$d\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}ds$$

Therefore, electric flux through the closed surface of the sphere,

$$\phi = \oint_s d\phi = \oint_s \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}ds$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\oint_s ds$$

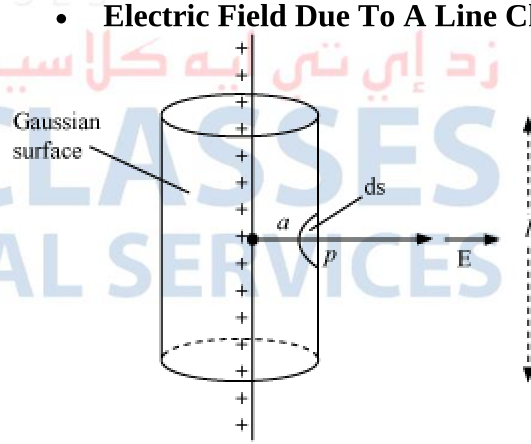
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\times 4\pi a^2$$

$$\therefore \phi = \frac{q}{\varepsilon_0}$$

It proves the Gauss theorem in electrostatics.

Applications of Gauss Law

- **Electric Field Due To A Line Charge**



Consider a thin infinitely long straight line charge of linear charge density λ.

Let P be the point at a distance 'a' from the line. To find electric field at point P, draw a cylindrical surface of radius 'a' and length l.

If E is the magnitude of electric field at point P, then electric flux through the Gaussian surface is given by,

$\Phi = E \times$ Area of the curved surface of a cylinder of radius r and	Because electric lines of force are parallel to end faces (circular caps) of the cylinder, there is no component of field along the normal to the end

length l faces.

$\Phi = E \times 2\pi al$ … (i)

According to Gauss theorem, we have

$$\phi = \frac{q}{\varepsilon_0}$$

$$\because q = \lambda l$$

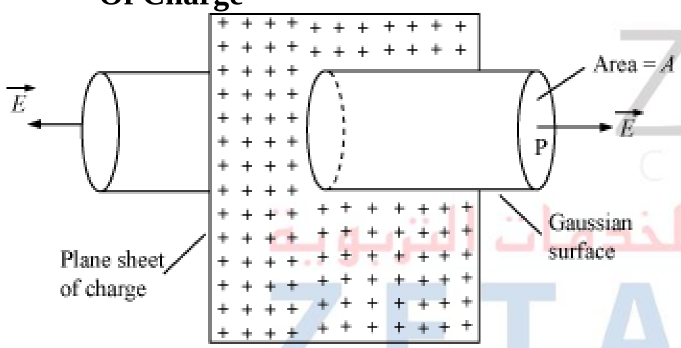
$$\therefore \phi = \frac{\lambda l}{\varepsilon_0} \qquad \text{...(ii)}$$

From equations (i) and (ii), we obtain

$$E \times 2\pi al = \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{a}}$$

- **Electric Field Due To An Infinite Plane Sheet Of Charge**



Consider an infinite thin plane sheet of positive charge having a uniform surface charge density σ on both sides of the sheet. Let P be the point at a distance 'a' from the sheet at which electric field is required. Draw a Gaussian cylinder of area of cross-section A through point P.

The electric flux crossing through the Gaussian surface is given by,

$\Phi = E \times$ Area of the circular caps of the cylinder

Since electric lines of force are parallel to the curved surface of the cylinder, the flux due to electric field of the plane sheet of charge passes only through the two circular caps of the cylinder.

$\Phi = E \times 2A$ … (i)

According to Gauss theorem, we have

$$\phi = \frac{q}{\varepsilon_0}$$

Here, the charge enclosed by the Gaussian surface,

$q = \sigma A$

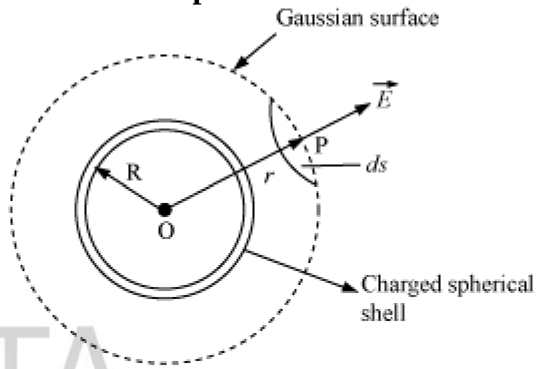
$$\therefore \phi = \frac{\sigma A}{\varepsilon_0} \qquad \text{...(ii)}$$

From equations (i) and (ii), we obtain

$$E \times 2A = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

- **Electric Field Due To A Uniformly Charged Thin Spherical Shell**



- **When point P lies outside the spherical shell**

Suppose that we have to calculate electric field at the point P at a distance r ($r > R$) from its centre. Draw the Gaussian surface through point P so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius r and centre O.

Let $\vec{E}$ be the electric field at point P. Then, the electric flux through area element $\vec{ds}$ is given by,

$$d\phi = \vec{E} \cdot \vec{ds}$$

Since $\vec{dS}$ is also along normal to the surface,

$d\Phi = E\,ds$

∴ Total electric flux through the Gaussian surface is given by,

$$\phi = \oint_S E\,ds = E\oint_S ds$$

Now,

$$\oint dS = 4\pi r^2$$

$$\therefore \phi = E \times 4\pi r^2 \qquad \text{...(i)}$$

Since the charge enclosed by the Gaussian surface is q, according to Gauss theorem,

$$\phi = \frac{q}{\varepsilon_0} \qquad \text{...(ii)}$$

From equations (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad (\text{ for } r > R)$$

- **When point P lies inside the spherical shell**

In such a case, the Gaussian surface encloses no charge.

According to Gauss law,

$E \times 4\pi r^2 = 0$

i.e., $= E = 0$ ($r < R$)